MATHEMATICS AND MUSIC: THE DELIGHTS OF PERFORMING MATHEMATICS

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"A mathematician, like a painter or a poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas."

"The mathematician's patterns, like the painter's or the poet's must be beautiful; the ideas, like the colours or the words, must fit together in a harmonious way. Beauty is Me first tow lea is no permanent place in the world for ugly mathematics."

G.H. Hardy

A Mathematician's Apology

The affinity between music and mathematics has been frequently the object of discussion. The mathematics of music, a topic covered in almost any introductory music book, has fascinated composers of all epochs, and many of the esthetic conventions of music can only be justified in mathematical terms. Musicians often feel that many of the more fundamental and deep properties of music are based on mathematics one way or another.

There are, on the other hand, some musical qualities to mathematics, something which, no doubt, is more difficult to explain. Esthetics plays an important role in mathematical research, and more often than not, the esthetics canons applied to mathematics are closer to music than they are to painting or sculpture. The reason some mathematical theorems are called beautiful has to do with some internal harmony, and with the way apparently unrelated ideas are connected.

Then is another way in which mathematics bears an affinity with music, and it has to do with the internal emotions each of these subjects produce in the person who experiences them and the way these emotions are transmitted. Music and mathematics are both created and performed; and through performance, conveyed to the public. This is not the case for painting which is created and exposed directly to the public it is true that the expert eye of a critic or another painter can see more in a painting than the eye of a layman, but the full creation of the painter lies there on the canvas, exposed to everyone.

Music is composed by an internal process of creation, and the result of this creation can be stored in codified form (by means of a score for example). But a musical piece can exist without being written, and brought out to the world each time it is performed. Something similar occurs with mathematics. A mathematical theorem is proved through an internal act of creation, the result is usually encoded in some form (in writing) and then it can be transmitted to others. But the mathematical result can be transmitted without being written, it can go from one mind to another through an explanation the same way music is transmitted through performance. A painting, on the

other hand, does not exist until it is painted.

Teaching a profound theorem to students in a class can produce the same kind of emotions a pianist might feet when performing in front of an audience: he feeds on the reactions the performance produces in the listeners. The emotion can be much more intense when the performer is playing his own compositions. A mathematician lecturing on his own work goes through the same kind of experience.

Sometimes outstanding mathematicians are not the best teachers, nevertheless it is a rewarding experience to listen to them explain their own creations; usually their lectures are full of "insight" even if they are disorganized or imprecise, Musicians conducting their own works or performing them are also, most of the time, extremely inspiring even if they are not accomplished performers.