Knowing, drawing, being: Is Mathematical Ekphrasis even possible?

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MISANU / Mathematics and Music Seminar

Mathematical Institute of the Serbian Academy of Sciences and Arts

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Three chapters (and a coda)

1 - Mathematical Ekphrasis?

2 - An Epistemological (Aesthetical?) Turn

3 - Ontology, the Sublime, Lyric Realism

Coda: Project Topoi/FourLogue

Framing Mimesis

Auerbach opens his Mimesis (written in exile in Istanbul during World War II) by a comparison of the Odyssey and the Abraham and Isaac passage in Genesis.

Whereas in the Odyssey descriptions of places, times, meals served in banquets, motivations and details (e.g. Odysseus' old scar, recognized by the old woman who worked for his family when he has come back, still unannounced) are detailed, in Genesis the story of Abraham leading his son Isaac to sacrifice is left bare:

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Whence? Beersheva? We don't know. Where is Moriah? Near Jerusalem? We don't know! They walk for three days, they leave in the morning. No details on what they eat, what Isaac says, what Abraham thinks.

Framing Mimesis

Yet, says Auerbach, by the power of its bareness, its barrenness, the story conveys an extremely powerful PLACE, a psychological PLACE, a place in consciousness still unfolding. . .

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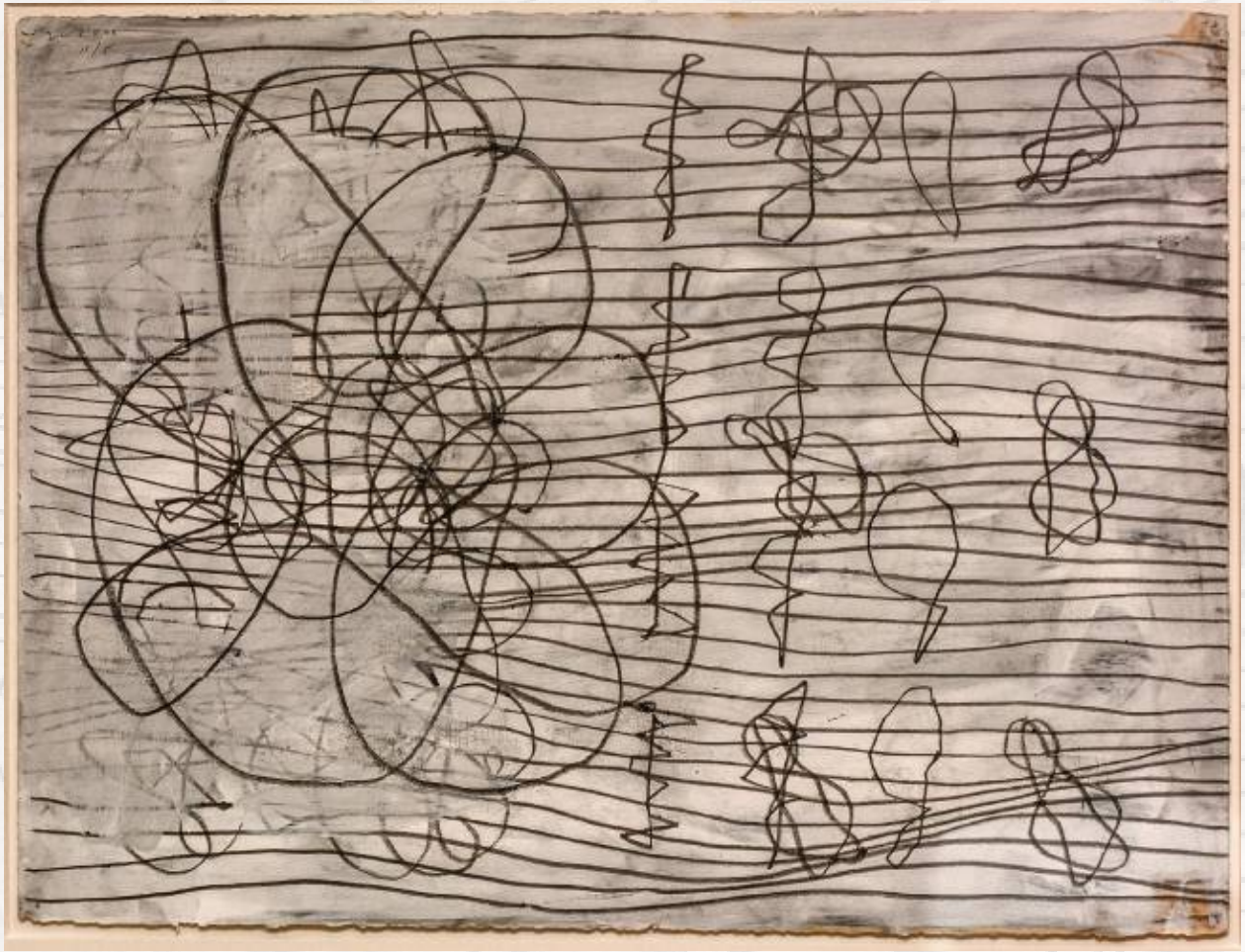
As in mathematical drawings...

Ekphrasis

But what happens between poetry and drawing?

Between mathematics and drawing?

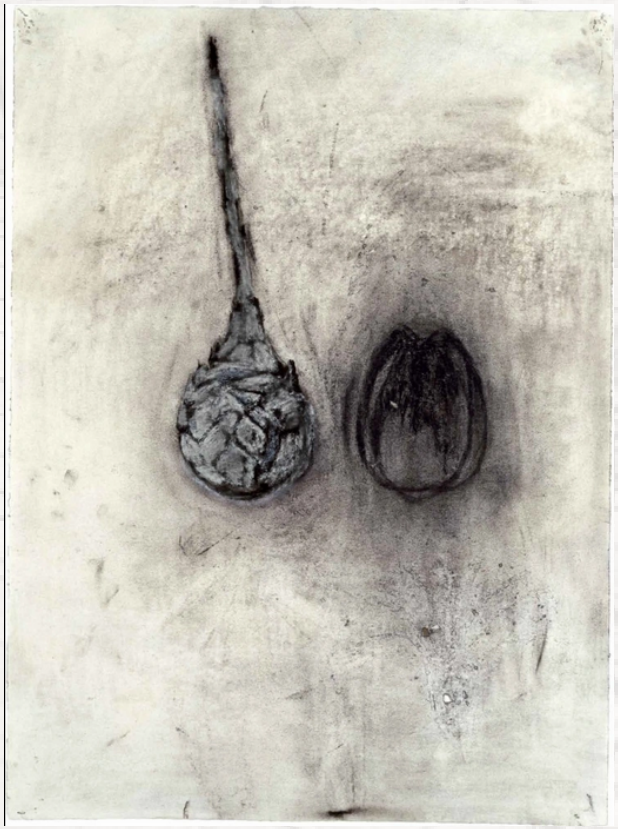
Peter Cole draws his poems from Winters' drawings



Peter Cole, on ekphrasis

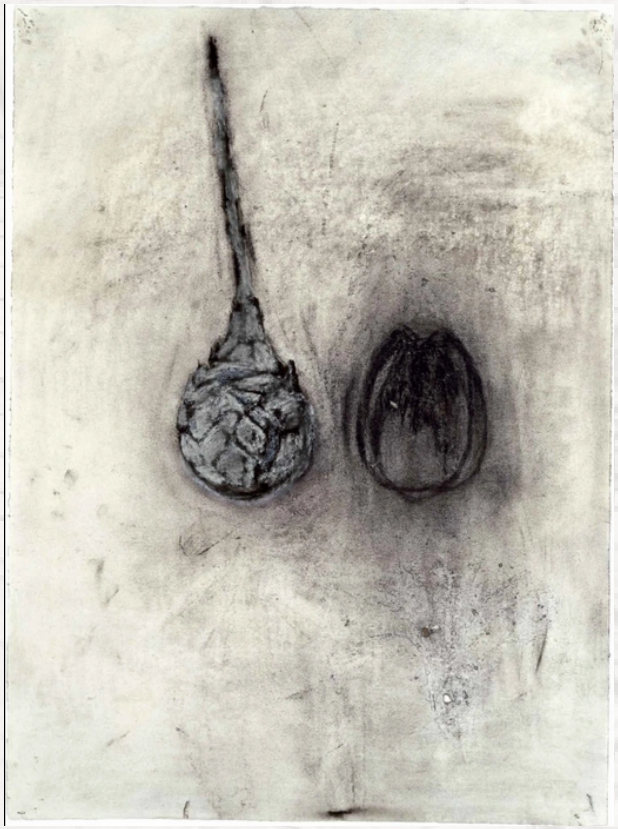
"They seem like knots, or a strange music's notes on a staff, in a whorl, like petals opening, to be heard. Something scored. Scars or sores. A soaring. Drawing words. It is the immediacy of ekphrasis that draws me. The contact. I realize it's odd to turn to another medium for a sense of immediacy. And yet, as with translation, that palpable sense of relation compels (completes?). And that's what I'm after — to speak into or through the drawings. To be dyed by their material qualities, as I feel them coming through me, or bringing me into their matrix. To take on their tinge."

Winters / Cole



"The smudged sounds give rise to lines, a syntax like synapses. Grappa in its capillary action. The narrow descent paradoxically widens out and lifts along a spectrum of endless adjacencies, in every direction, and every inflection,' as Levi Yitzhak of Berditchev sang, in his gentle Yiddish, 'Still You. However You. Only You. Every You . . .'"

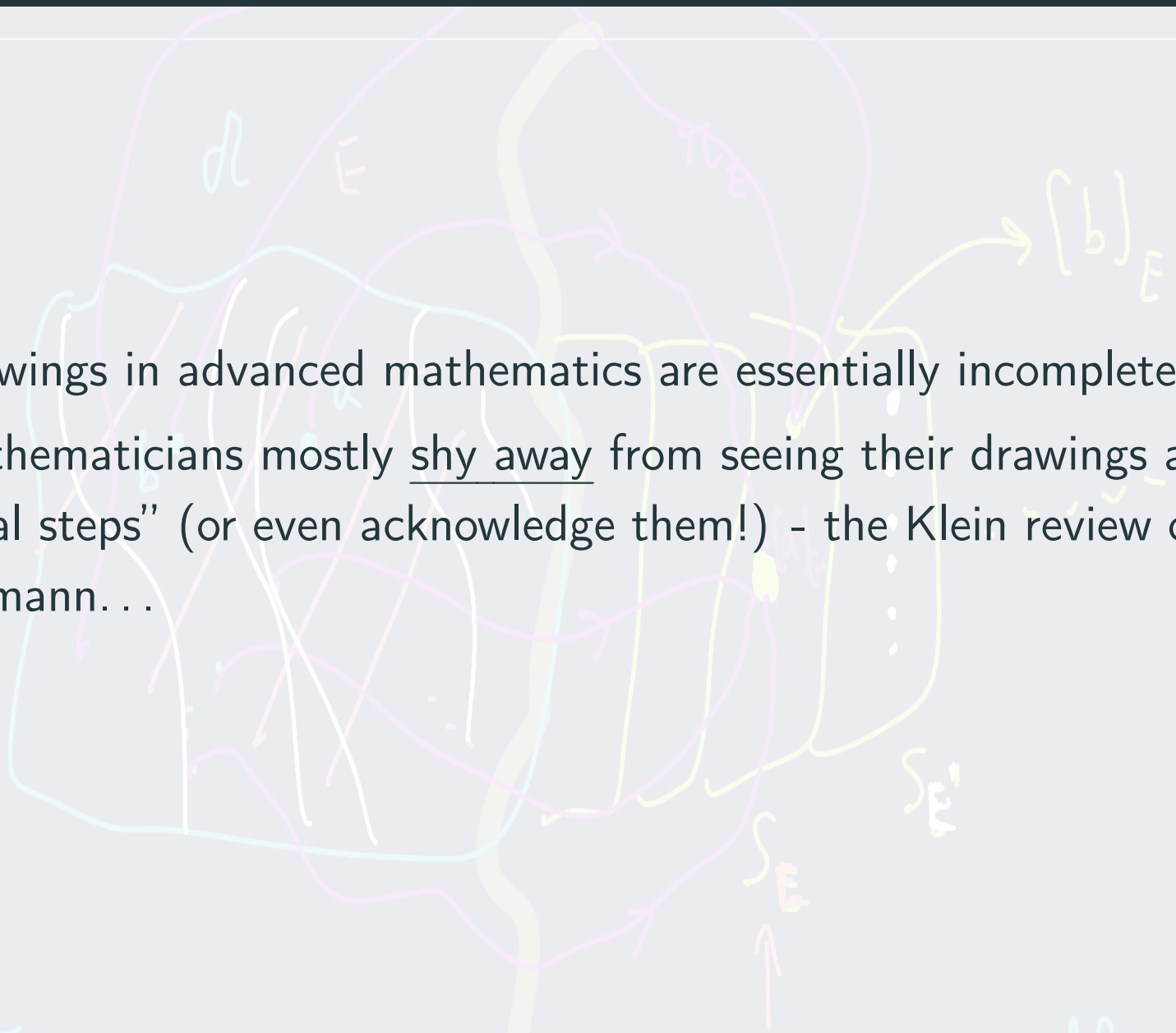
Winters / Cole



"You, the viewer? The reader? Whoever you are, and where . . . Drawing really does — draw us in (to the object rendered and the time taken) and out (of ourselves to further seeing and other surfaces, even souls, or simple tensility sensed)."

Peter Cole - On Being Drawn

Essential incompleteness + Shying Away from Showing: two axioms?



- Drawings in advanced mathematics are essentially incomplete
- Mathematicians mostly shy away from seeing their drawings as "final steps" (or even acknowledge them!) - the Klein review of Riemann...

Back to the problem - some issues

Peter Cole evokes the closeness to death in the act of translation, and makes it an essential part of drawing.

Syntax like synapses truly seems to cut to the heart of drawing.

Capillary action (capillary mathematics?). The narrow descent (of what? lymph? a vital fluid? information?) and the adjacencies, in every direction, and every inflection (Leibniz-like theme here?)

The You from Levi Yitzhak of Berditchev, possibly the viewer, the reader, the mathematician who hears a proof explained.

Back to the problem - Coda (of the Overture)

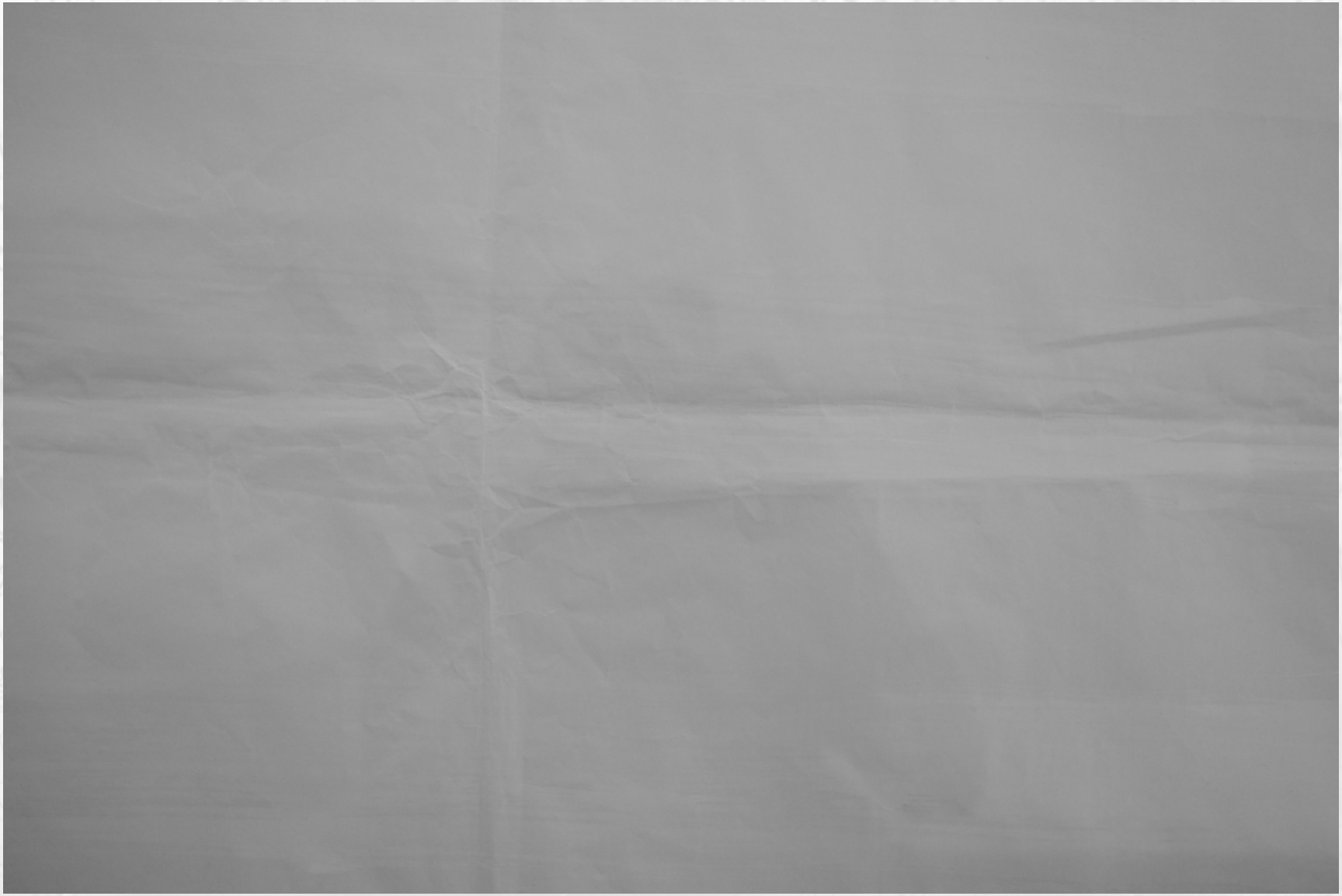
And drawing draws us in and out (to the object rendered, the time! and of ourselves to further seeing)... Drawing and undrawing, drawing and erasing constantly, capillary action. How many times does a mathematician draw/redraw/undraw/redraw a proof until she sees it? Until she is drawn in-to the proof/object and the time taken, until she is drawn out to other surfaces? What is tensility in mathematics? What is capillarity?

Ekphrasis?

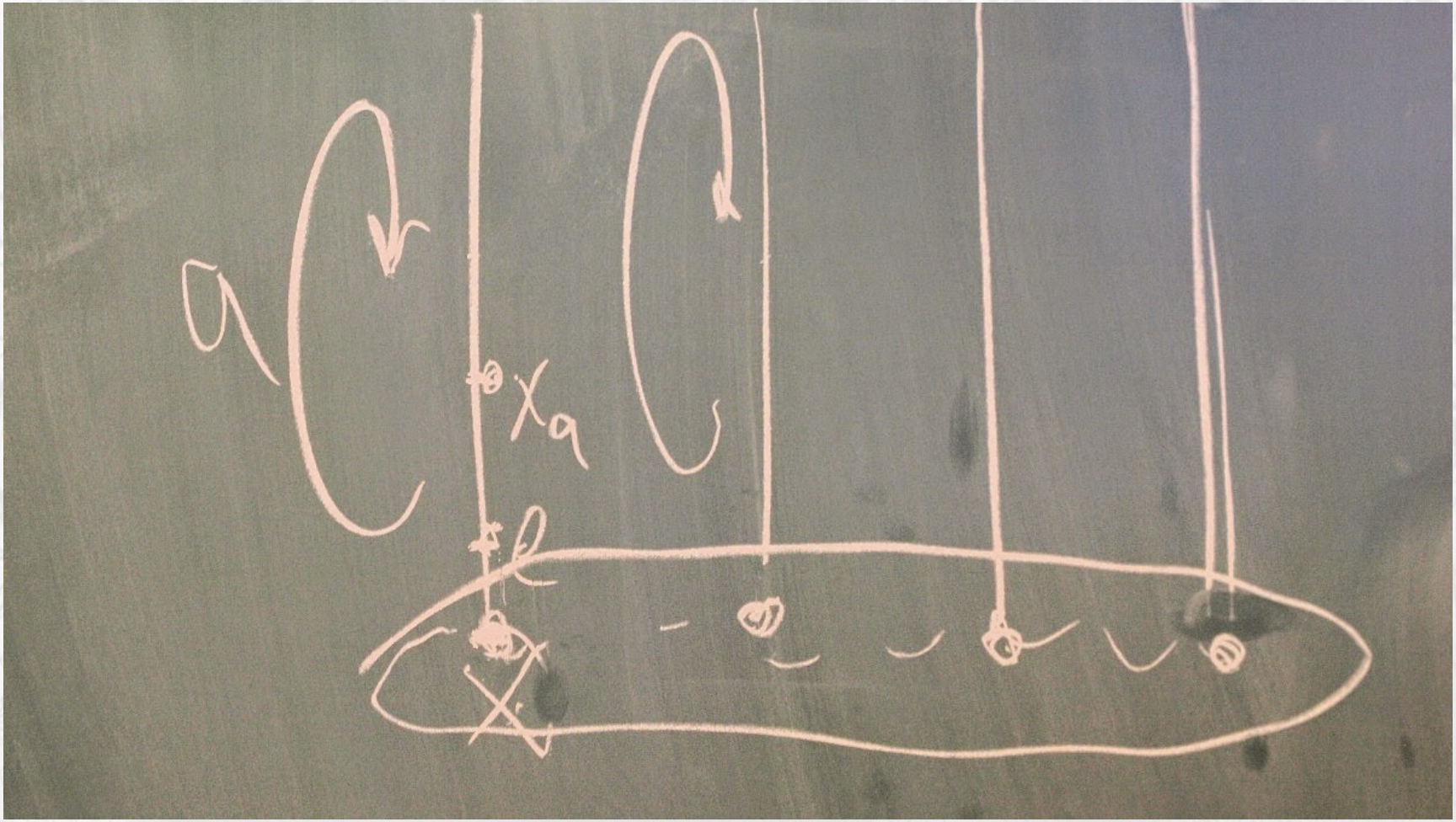
Is mathematical ekphrasis even possible?

How does grounding operate between mathematics and its drawings?

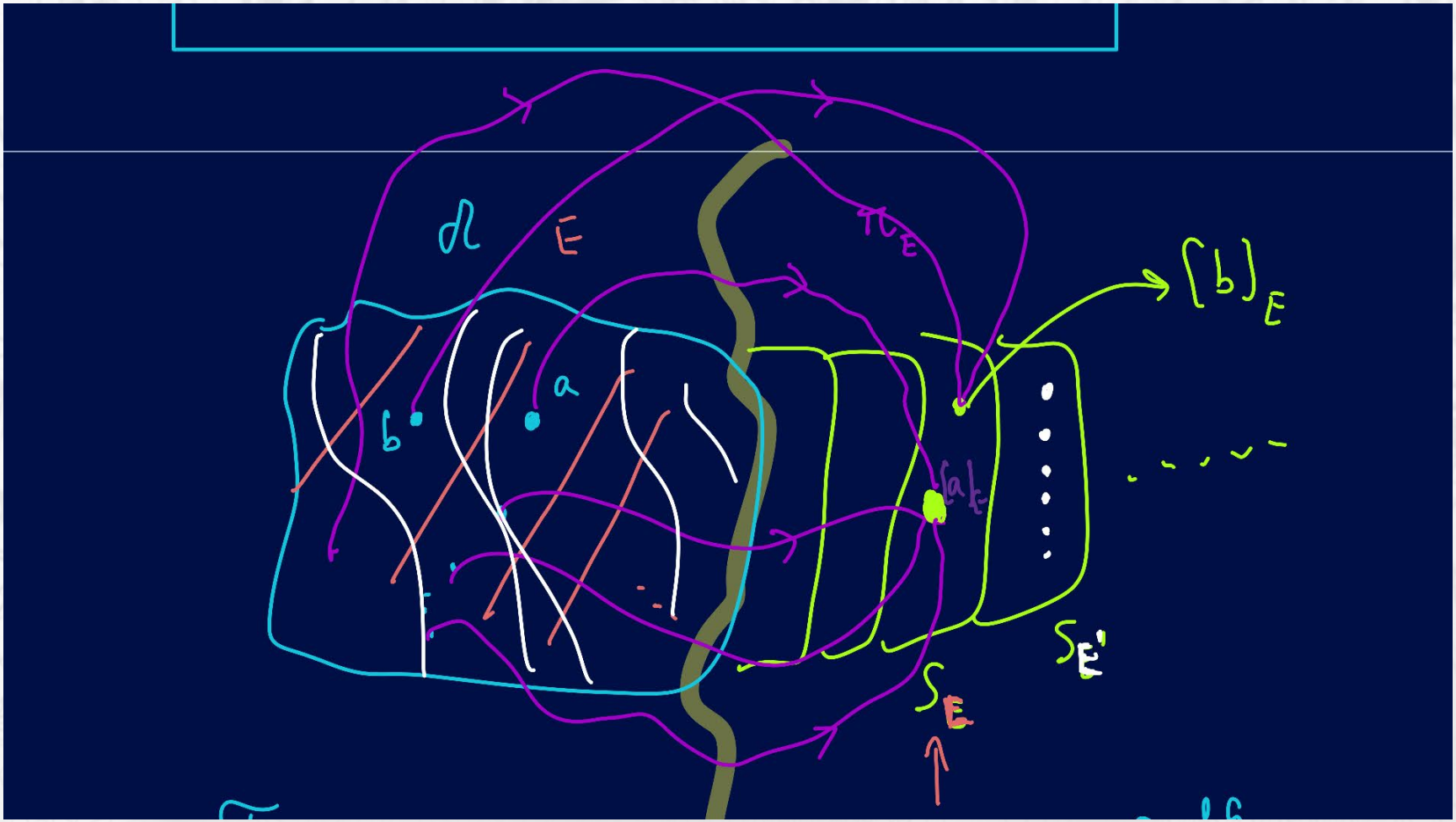
A small gallery / A fold by K. Hilten



A small gallery / B. Zilber (Zariski geometry/physics)



A small gallery / A.V. (Imaginaries in model theory)



A small gallery / A.V. (Unfoldable cardinals)

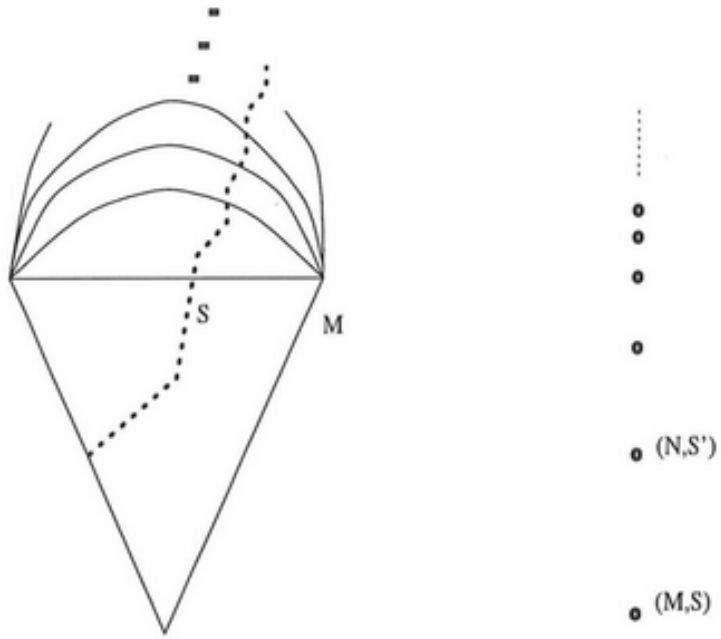
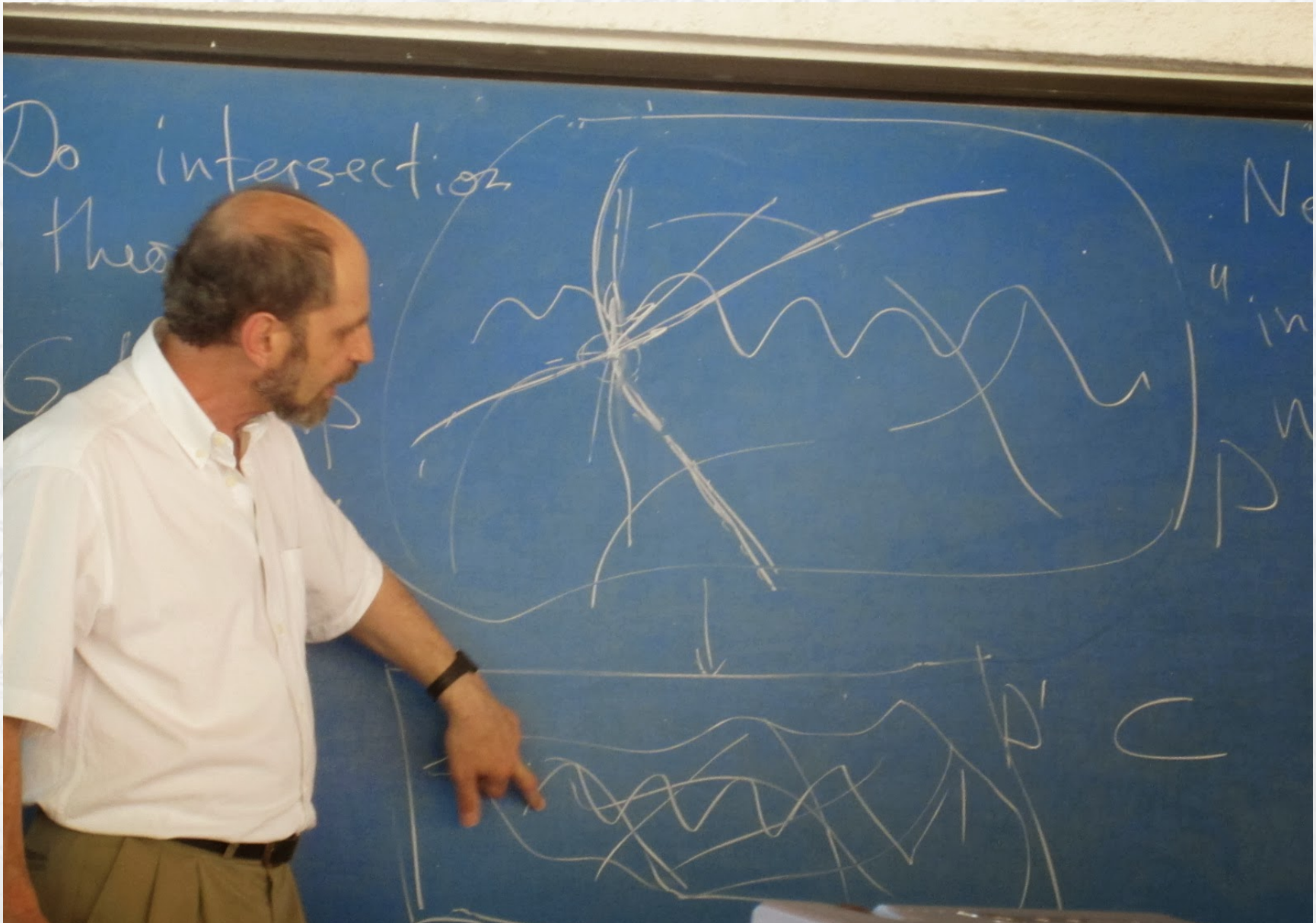


Figure 3: Unfoldables and Chains in $(\mathcal{E}_{(\mathcal{R}(\kappa),\in,S)}, \prec_e)$

The parameter S in the definition of unfoldable cardinals is crucial for them to be of any interest: as Enayat points out in [6], given any model M of ZF such

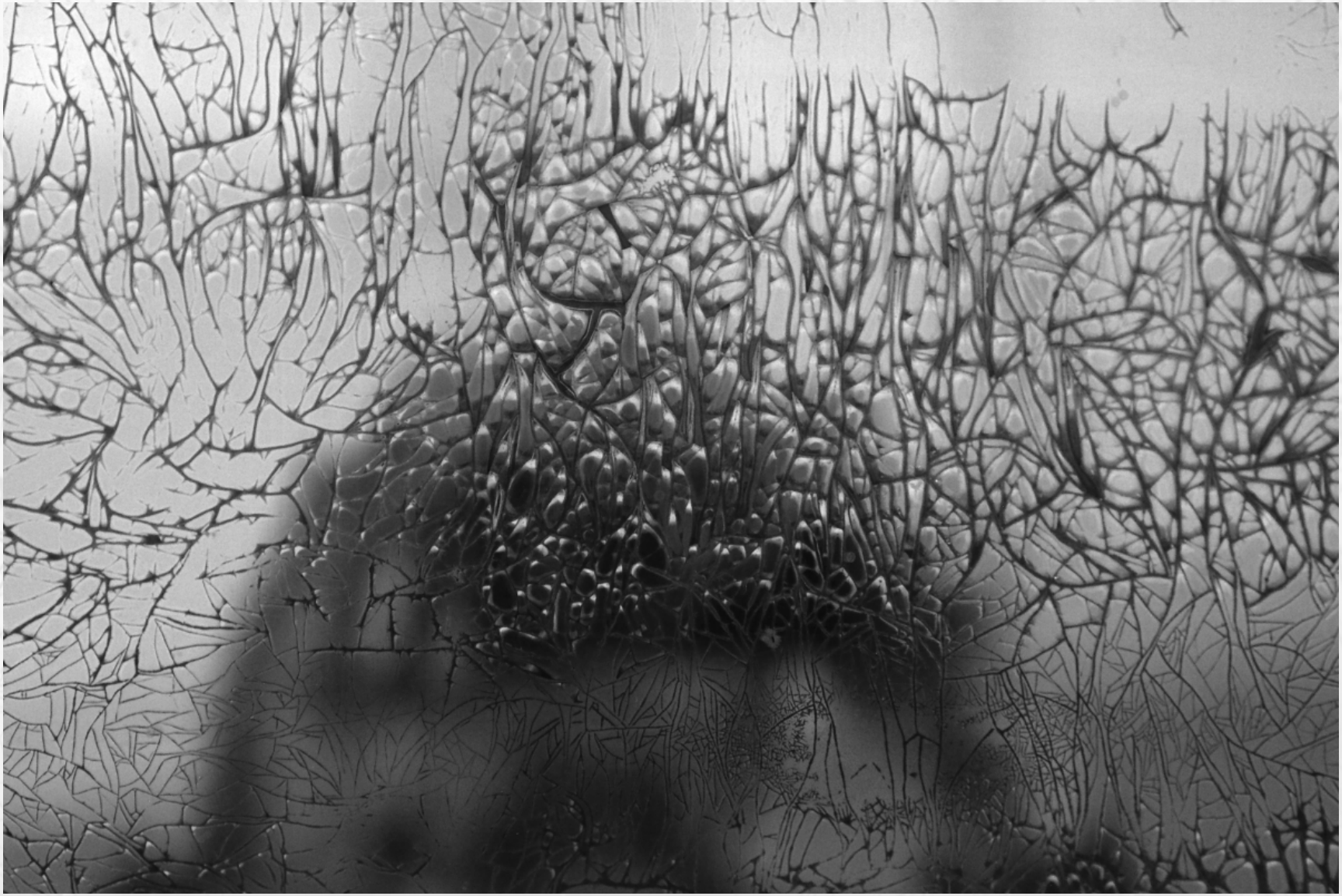
A small gallery / B. Zilber (Trichotomy and Quantum)



A small gallery / Lartigue (Grises)

A small gallery / Colombian Amazonia, 14000 years ago

A small gallery / Hilten



A small gallery / F. Sandback

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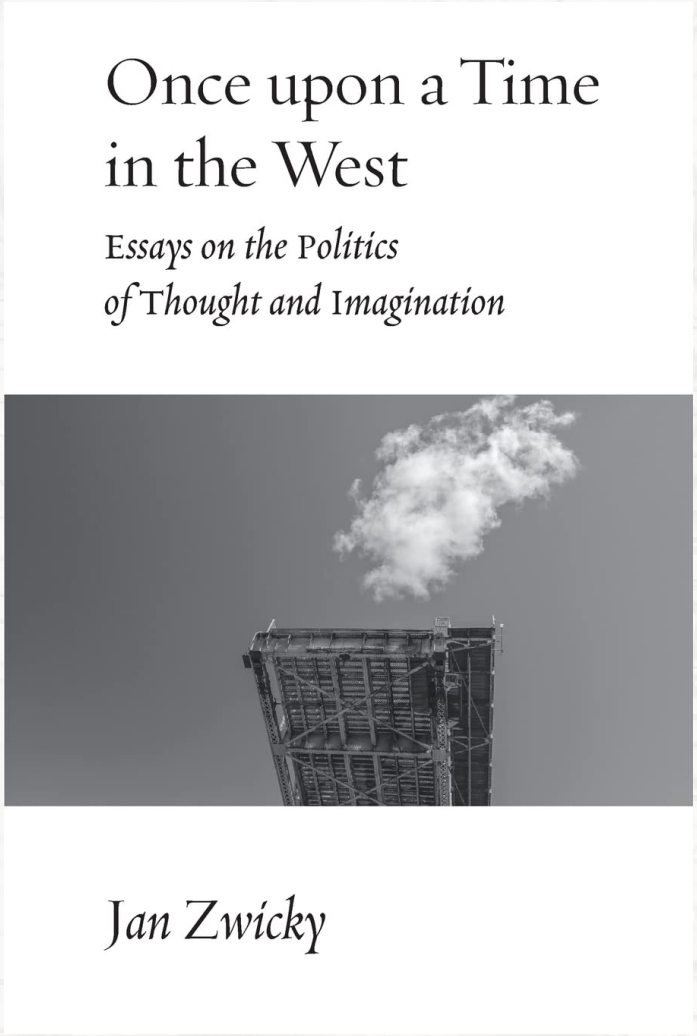
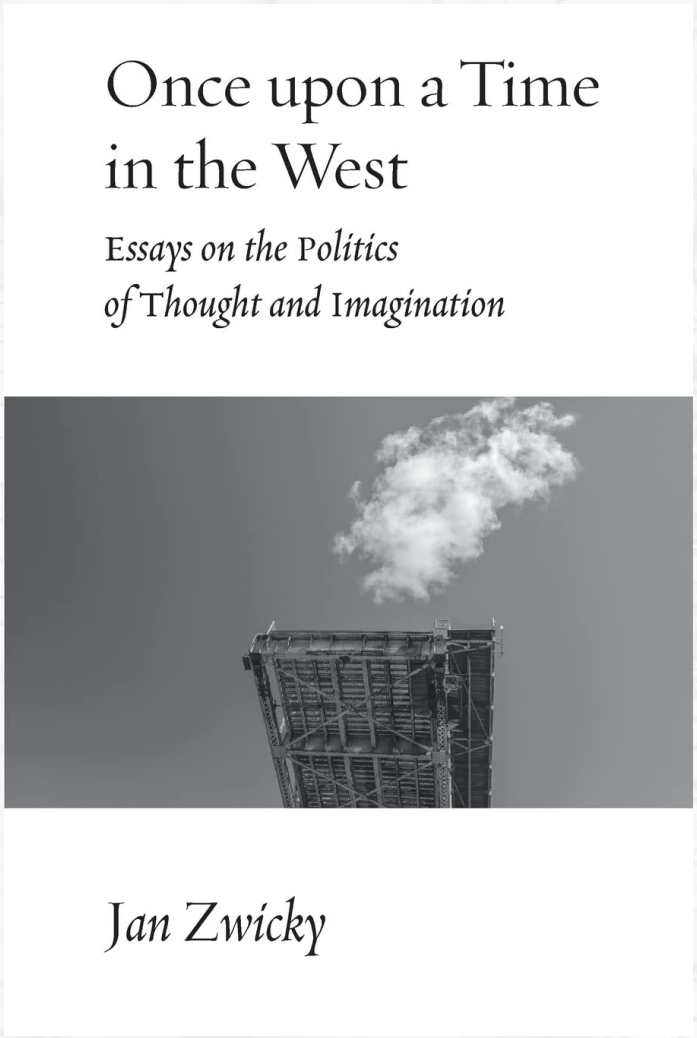
Auden's Question: How Poets Think?



In her recent (2023) book **Once upon a Time in the West**, Jan Zwicky frames Auden's question ("How a poet knows what he knows") from his 1956 inaugural address at Oxford as the formulation of an epistemology, an account of lyric comprehension.

Auden's Question: How Poets Think?



The core of Auden's theory is the faculty of **imagination.**

seeing as / hearing as

(as in mathematics and music, we recognize melodies and proofs, beyond changes of tonality or different (even incomplete) presentations)

imagination as "bringing to light"

Auden: the objects of imagination *oblige* us to respond. Imagination *perceives things that exist.*

Zwicky's reading of Auden's epistemology

Zwicky describes Auden's epistemology in four parts:

- Primary Imagination
- Secondary Imagination
- The origin of the work of art
- His evidence that we should believe it...

Primary: perception of sacred beings and sacred events (moon, fire, darkness, nothing, death...) - a realm without freedom, self-forgetful, with no sense of time, pre-linguistic!

(Freud - Peirce - Kant (through Schelling and Schiller) - **Coleridge**)

Secondary: active, discerns fitting means of expression for the awe given by (1)

Wordy/wordless awareness in making poetry (mathematics?)

Zwicky: *some writers (. . .) focus almost entirely on the Secondary Imagination: the linguistic skin of the poem [stuff or skin?] (. . .) Such writing does not make the impression on me that what I call 'poetry' does (. . .) poetry is the kind of writing that has that non-linguistic 'interior'*

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The **non-linguistic interior** seems like a strong topological/epistemological tenet, applicable to mathematics!

In 2022, French algebraic topologist David Bessis wrote a whole book (**Mathematica**) based on the role of dreams (notably in Descartes, Einstein and Grothendieck) in the formation of the mind of mathematicians!

Auden vs Bacon: two conflicting epistemologies

Auden's Poetic Epistemology	Epistemology of Baconian Science
Epistemic duty: *To praise being*	Epistemic duty: *To judge* (T/F)
Epistemic attitude: *Gratitude*	Epistemic attitude: *Suspicion*
Purpose: *None*	Purpose: *Control*
Organ: *The imagination*	Organ: *The mind*
Litmus test: *Awe, Fitting*	Litmus test: *Systematic argument*
Evidence (genuine paradigm): *Cultural need of poetry*	Evidence (genuine paradigm): *Success of Science and Techn.*

Mathematics really seems to be closer to poetry than to 'Baconian' science

Yet...

Syntax / Semantics

Syntax / Semantics

Epistemology or Aesthetics?

Syntax / Semantics

Epistemology or Aesthetics?

Again, similar pendularities!

In contemporary mathematical logic, two directions:

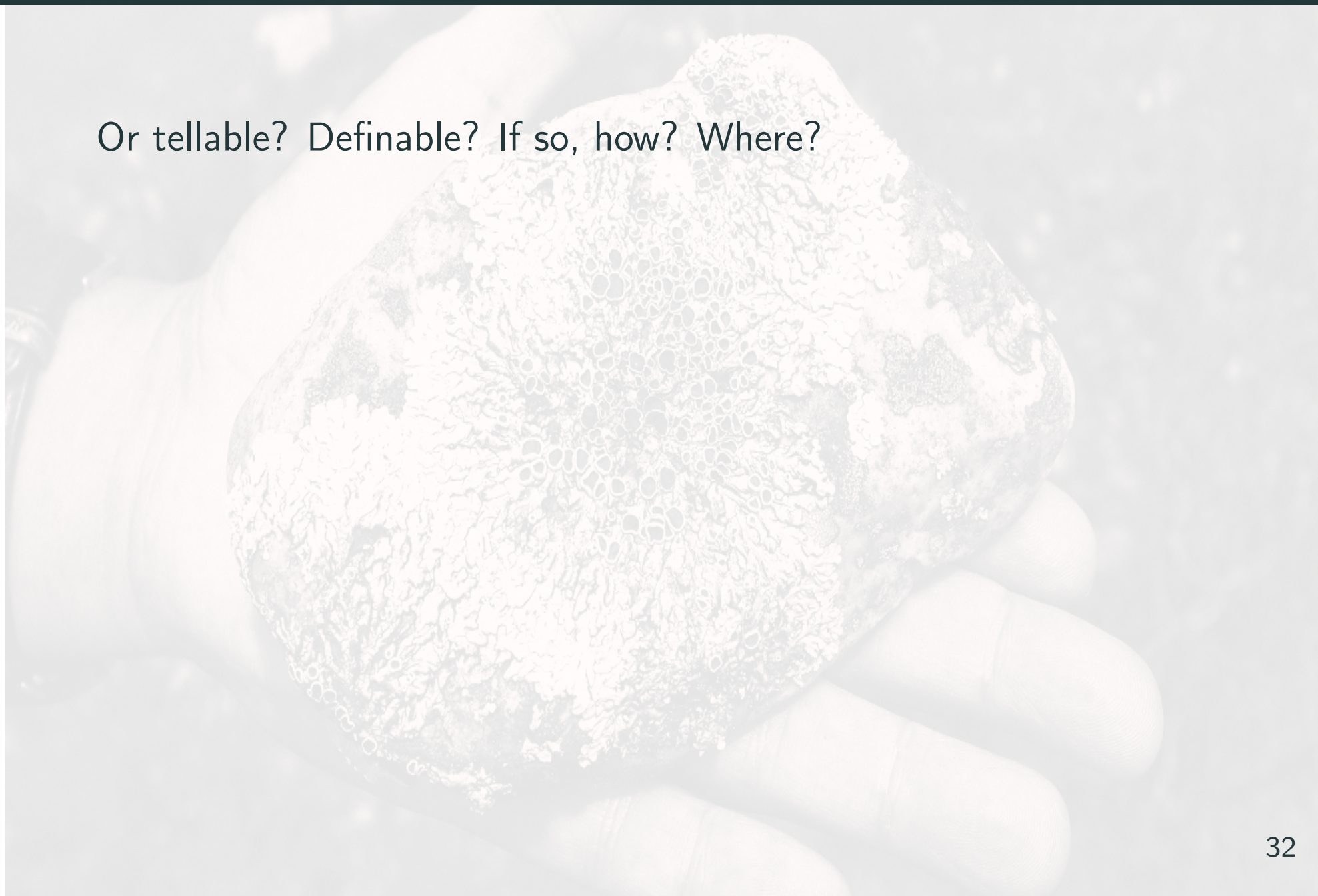
- Inner Models from Extended Logics (Kennedy, Magidor, Väänänen) - extracting (robust) meaning from language?
- Abstract Elementary Classes and the Presentation Theorem + Recent Axiomatizations (my joint work with Shelah): extracting language from (robust) meaning?

Aesthetics and Logic? Grasping!

Zwicky in her Plato as Artist follows the dialogue Meno with an ear to the interplay between Aesthetical, Morality and Phenomenology. Her attention to the problem of grasping as alluded to by Plato/Socrates in that dialogue, in response to the question Is virtue teachable? points toward some of the issues we are discussing today, between aesthetics and logic.

Meno: Is virtue (or anything else) teachable?

Or tellable? Definable? If so, how? Where?



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Grasping

Logical “eye” vs Geometrical “acting”

We may “ask questions” to structures: can you see the tower behind you in a Renaissance painting? is it ok to resolve your sonata in C major if you started in D minor? Can we travel back to yesterday? If so, how?

We then

compare - augment - diminish - stretch - shorten - ...

our structures.

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Model theory is the mathematical theory that studies in full generality these possibilities - it is naturally anchored in logic, in the possibility of querying a structure, in the implicit language it supports.

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Model theory provides the building blocks, the "primary colors" of structures

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Model theory is the mathematical theory that studies in full generality these possibilities - it is naturally anchored in logic, in the possibility of querying a structure, in the implicit language it supports.

Model theory provides the building blocks, the "primary colors" of structures
then blends them, helping produce all possible "colors", all possible structures, and...

Yet, surprisingly

in recent years - after Model Theory sharpened its own logical seeing to the point of providing a classification of all possible (first order) theories and asymptotic dividing lines (the Main Gap), it embarked itself into a second sailing, towards the side of "action", towards geometry, apparently away from logic!

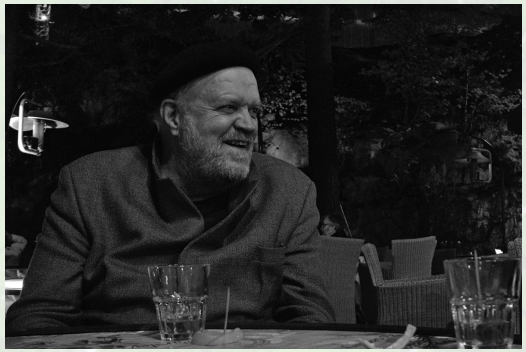
But really? Emphasis in *fitting*

> *"We have in mind mainly those interested in algebraically-minded model theory, i.e. in generic models, the class of existentially closed models and universal-homogeneous models rather than elementary classes ..."*
>
> Saharon Shelah, 1975.

An enormous development of model theory started there, essentially pushing aside the centrality of logic, and pushing to the front abstract notions of "fitting" ($M \prec N$): **abstract elementary classes** are **coherent, smooth, forward-continuous generative systems.** They largely replace (and wildly extend) first order logic. They resolve "inverse Galois problems" of algebraic topology of categories.

Inner Models for Different Logics





Emergence of freedom from formalism can be seen in the work of Kennedy, Magidor and Väänänen: they adapt Gödel's definable hierarchy to other logics, and end up studying naturality (and set-theoretic spectra) of logics through the behaviour of the inner models obtained.

Marcolli-Berwick-Chomsky: LLMs and Hopf Algebras

A linguistic turn: very recently (November 2023) Marcolli, Berwick and Chomsky have announced a **syntax-semantics interface** for Merge and Minimalism linguistic models, based on Hopf Algebras (they adapt ideas from renormalization in physics; especially in quantum field theory). They address controversies/misunderstandings between generative linguistics and LLMs (Large Language Models) (https://www.youtube.com/watch?v=-gx3SK7FvKk)

SYNTAX-SEMANTICS INTERFACE: AN ALGEBRAIC MODEL

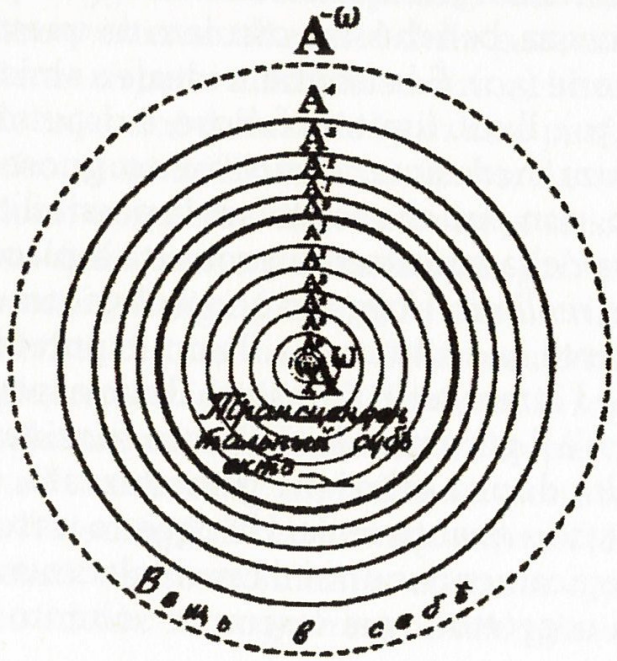
MATILDE MARCOLLI, ROBERT C. BERWICK, NOAM CHOMSKY

ABSTRACT. We extend our formulation of Merge and Minimalism in terms of Hopf algebras to an algebraic model of a syntactic-semantic interface. We show that methods adopted in the formulation of renormalization (extraction of meaningful physical values) in theoretical physics are relevant to describe the extraction of meaning from syntactic expressions. We show how this formulation relates to computational models of semantics and we answer some recent controversies about implications for generative linguistics of the current functioning of large language models.

CONTENTS

Florensky's time-generating (and reversing) epistemology

cacemente dal seguente schema:



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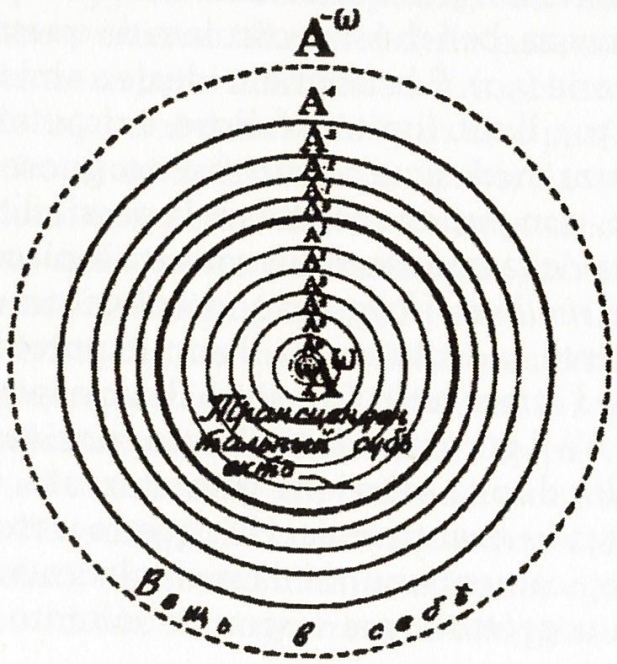
Knowledge as an act of consciousness of not being non-knowledge.

Discerning = knowledge

Florensky's time-generating (and reversing) epistemology

cacemente dal seguente schema:



48

- Mere knowledge (A_1), knowledge of knowledge (A_2).
- Iteration: third level (A_3), fourth level knowledge A_4, and so on ad indefinitum.

$$A_1, A_2, \ldots, A_n, A_{n+1}, \ldots$$

- A as an operator: $A_1 = A^1 = A(1)$. 1 symbolizes the object, A the act of isolating i from the rest
- Comparing acts:

$$\ldots = \frac{A_n}{A_{n-1}} = \frac{A_{n-1}}{A_{n-2}} = \ldots = \frac{A_2}{A_1} = \frac{A_1}{1}$$

- Negative powers $A^{-1}, A^{-2}, \ldots, A^{-n}, A^{-n-1}, \ldots$ enact nicosophy (as opposed to philosophy...)

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Some directions for mathematical ontology(ies)

In this short section, I briefly touch four different proposals connected with mathematical ontology, only as a (provocative?) foretaste of possible discussions for these times:

- Zwicky's *lyric realism*
- Kennedy's *mathematical sublime*
- Badiou's extreme *equation* (and its roots in Novalis)
- Zalamea's blend of epistemology, phenomenology and ontology in the *rTHK model*

Zwicky: Lyric Realism

In *Lyric Realism: Nature Poetry, Silence and Ontology* (2023), Jan Zwicky describes lyric thinkers (and the special case of nature poets) in terms of *understanding driven by intuitions of coherence*.

She offers abstract definitions of **nature** (driven by the etymological roots *nascī* and the connections with *kin/ĝen*, kind:

nature is the tendency of things to be what they are, and in that tendency to present themselves as both distinct and connected

. . . the working out of origins through individuals

The Mathematical Sublime (Kennedy))

In *Boris Zilber and the Model-theoretic Sublime* (2023), Juliette Kennedy starts from descriptions offered by the mathematician (model theory of geometry) Boris Zilber (professor at Oxford) on syntax-semantics tensions running through his long-winded project.

She zooms in to ontology when discussing **place**, the role of this category in Zilber's work: *Is anything real in mathematics, that is* not *related to geometry? "Nothing is that is not placed", as Plato has reportedly said."*

Then she contrasts with J. Floyd's notion of surveyability and offers the new category of the **model-theoretic sublime** as an aesthetic framework providing comparisons with (e.g. the ecological sublime, the romantic sublime, the technological sublime, etc.) studied by E. Apter.

Novalis-Badiou's Mathematical Ontology

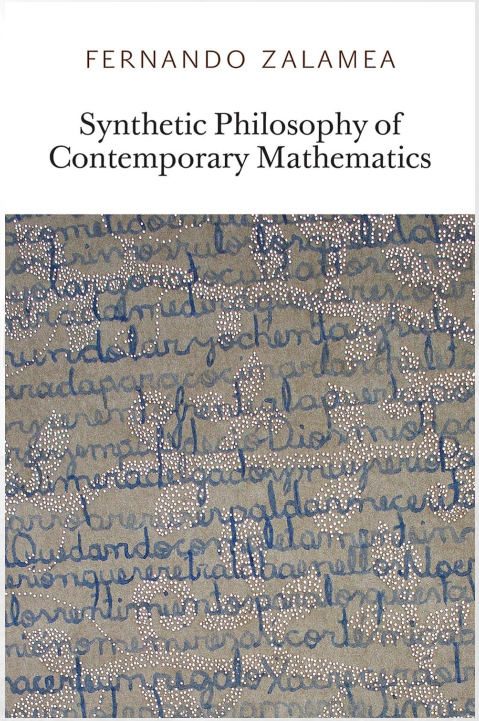
In a volume dedicated to the 40 years of Badiou's Being and Event, edited by M. Dzamonja and T. Tho, I addressed (*Ontologies étalées*) the radical equation opening Badiou's book

$$\textbf{mathematics} = \textbf{ontology}$$

and its further development along the history of set theory; part of the beginning of that paper deals with an interesting (and surprising) rooting of Badiou's ontology in Novalis's description of **being coming from emptiness.** Novalis's ontology contains the complete kernel of Badiou's radical equation, two hundred years *avant la lettre!*

Zalamea's Synthetic Philosophy (and rTHK model)

In 2010, Zalamea proposed a first model blending epistemological, phenomenological and ontological approaches to mathematics, and he provided there the categories of *archeal mathematics*, *quiddital mathematics* and *eidal mathematics*. This was one of the main content of his Synthetic Philosophy of Contemporary Mathematics. Then, in 2022 he extended his model to the **rTHK** construction (Riemann, Topoi, Sheaves, Kripke) (*Modelos en haces para el pensamiento matemático*).



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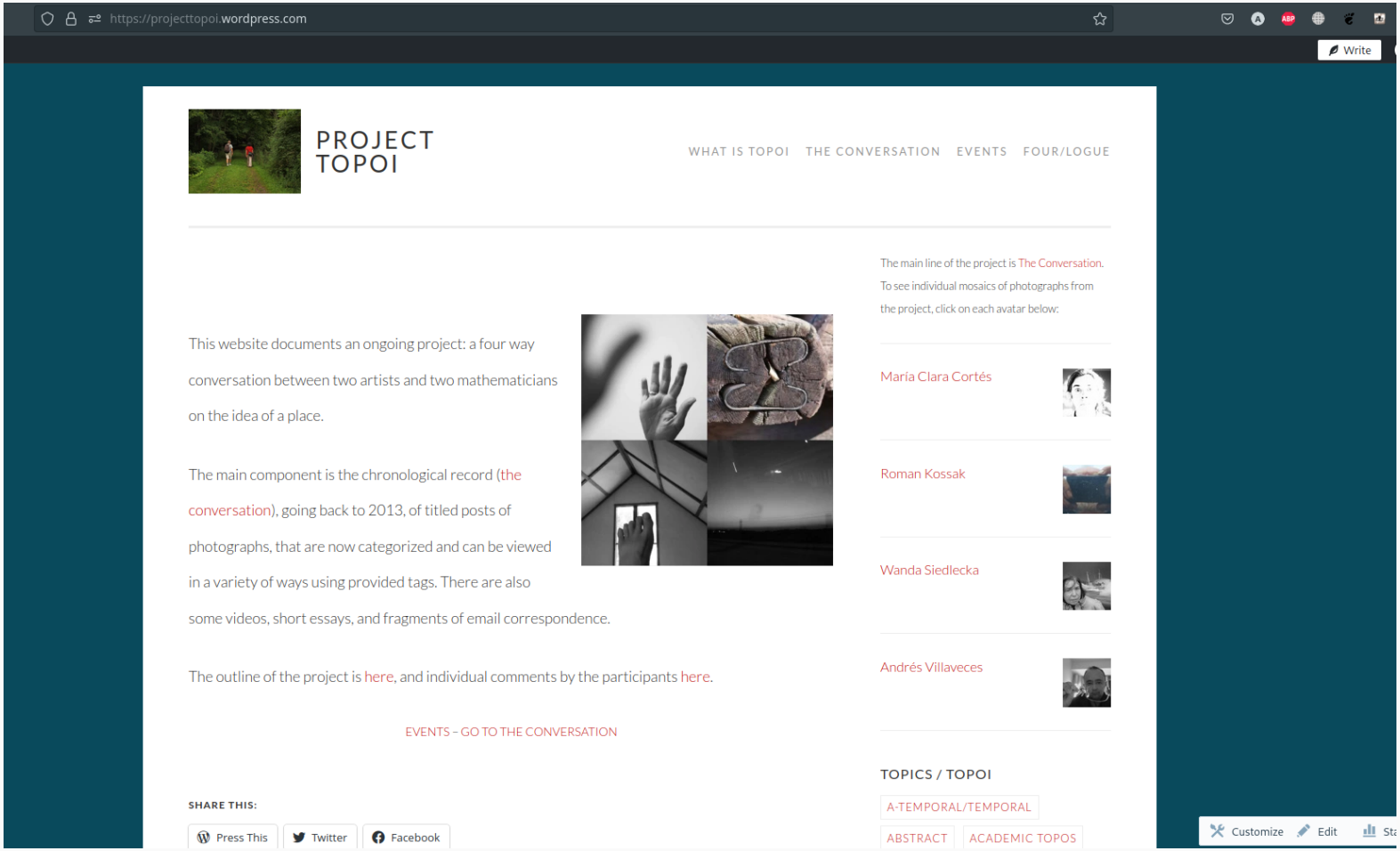
In 2013, several failed attempts at communication with artists had led the mathematical logician Roman Kossak (then at CUNY) to write another paper on structure and art (thereby trying to enact ekphrases of various kinds).

A visit to Colombia reframed and recast completely the attempts of a conversation, and we started a project: artists Wanda Siedlecka and María Clara Cortés, mathematicians Roman Kossak and AV - we started a dialogue purely based (at first) on photography, trying to find **topoi**, PLACES, hopefully common or at least visible to one another.

The power of non-verbal communication enabled (between 2013 and 2016) a very rich dialogue.

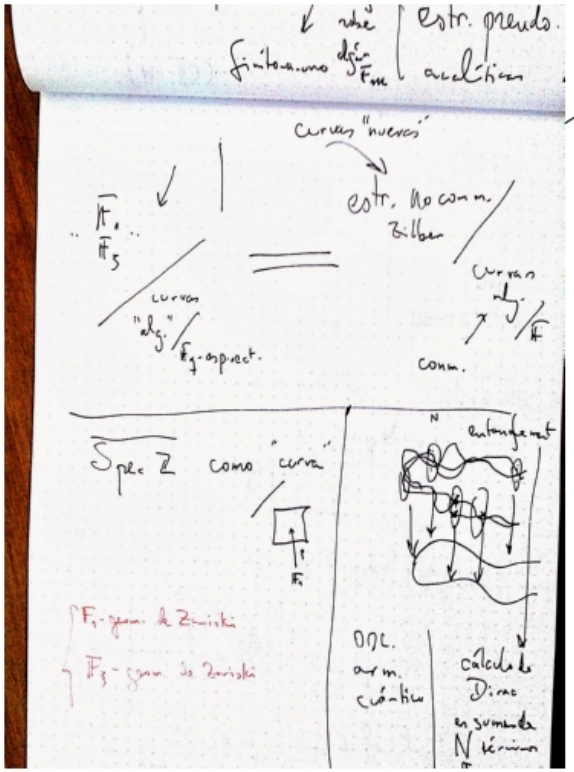
The project restarted after the pandemic, with different aims and styles (FourLogue).

Project Topoi (https://projecttopoi.wordpress.com/)



Coda: Project Topoi

Proyecto **moving topoi**



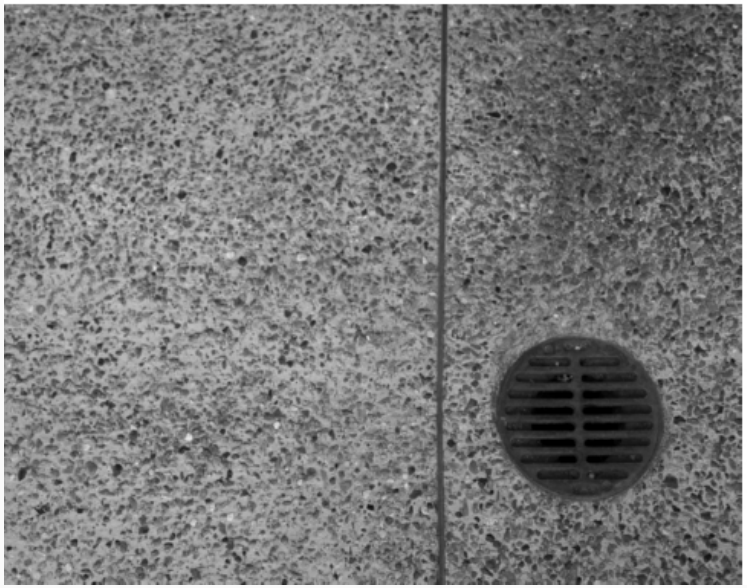
MAT

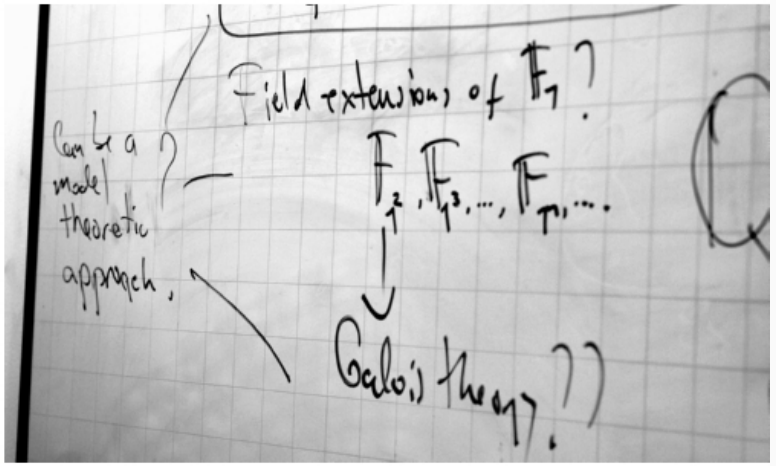
?

ART

¿Un diálogo imposible?

- Creación de mundos
- Percepción del mundo
- Atrapar movimiento
- Develar estructura
- Sueño/vigilia
- ...





Barreras fuertes de Comunicación...

Cuentos de Tono : Moriyama

Matemática de los últimos 50 años

FEN (FIL)

Samuel TODES : el **mundo** de percepción está hecho de campos dentro de campos...

topoi

Project Topoi

Capturar espacio

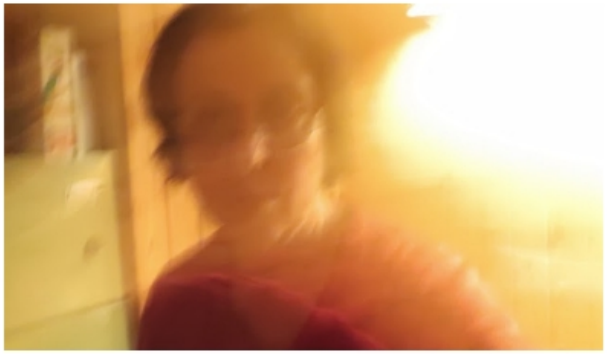
/

Explorar
paralelismos
y dicotomías

¿topoi matemáticos
y
artísticos?

Project Topoi

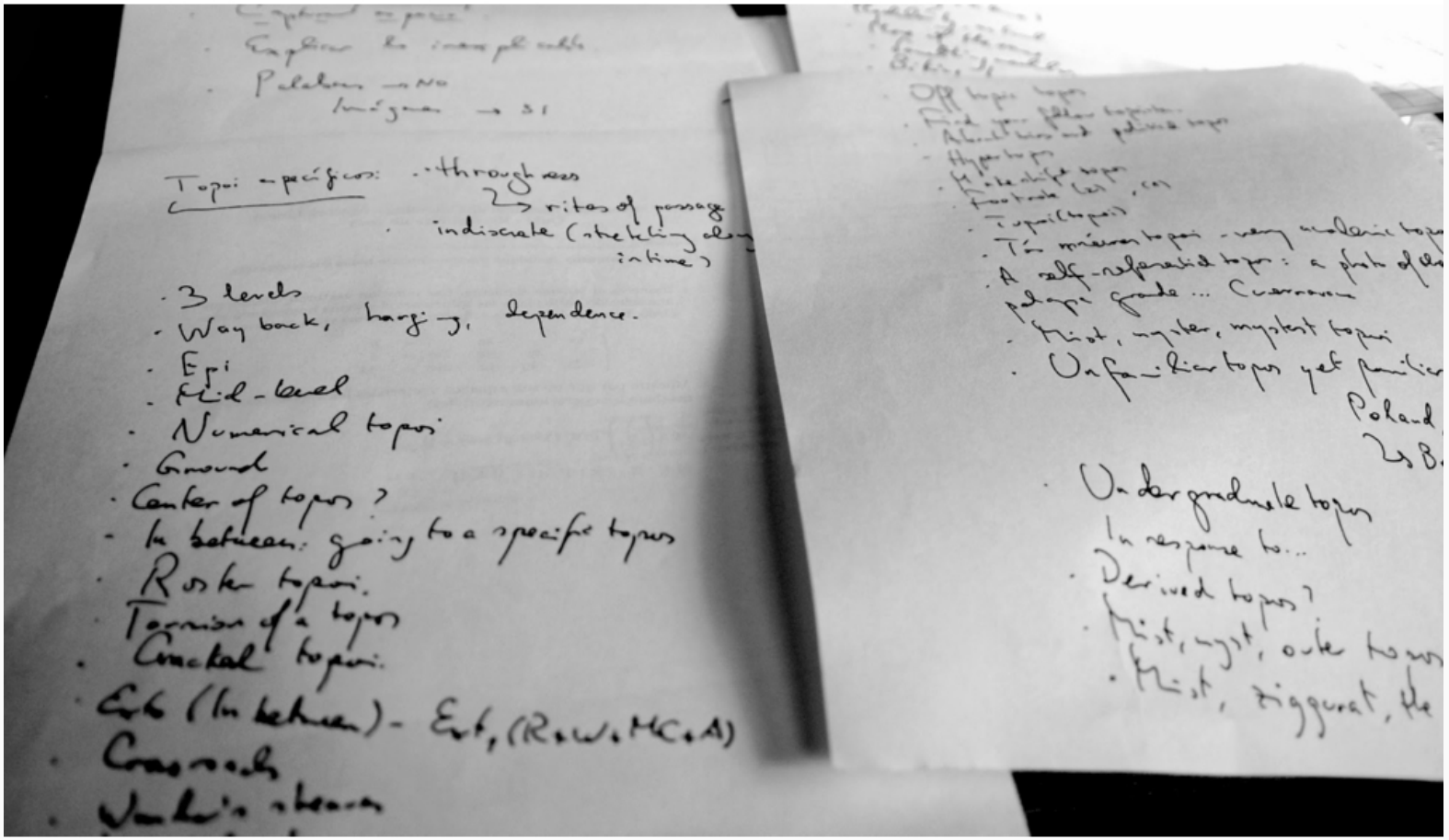
María Clara **Cortés**, Wanda **Kossak**,
Roman **Kossak**, Andrés **Villaveces**



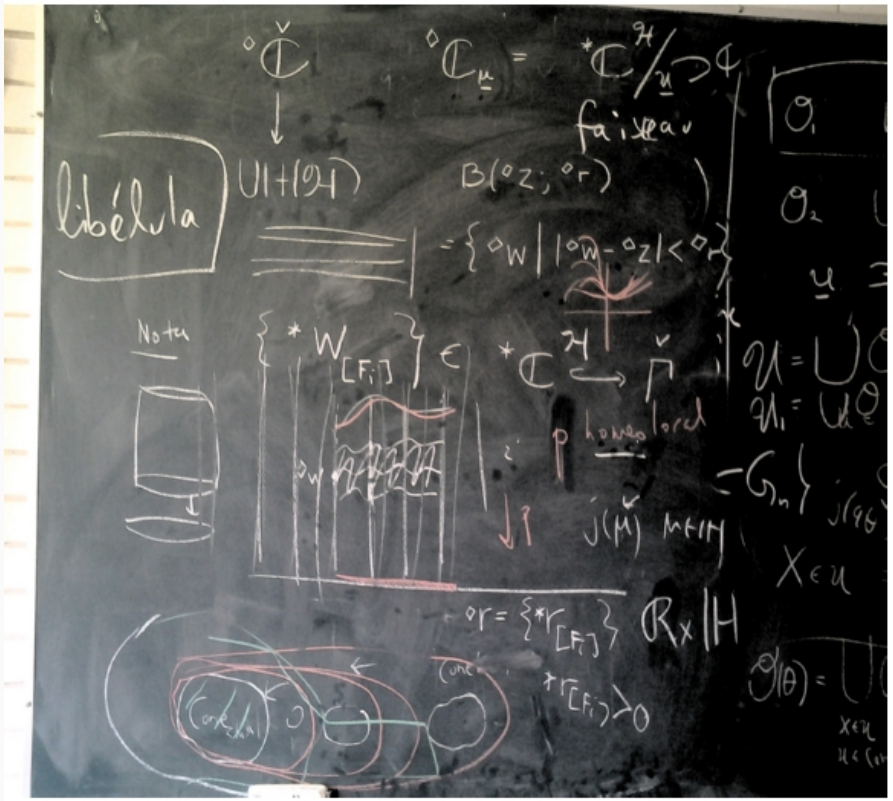
Dicotomías - $(2 \cdot 2)^n$

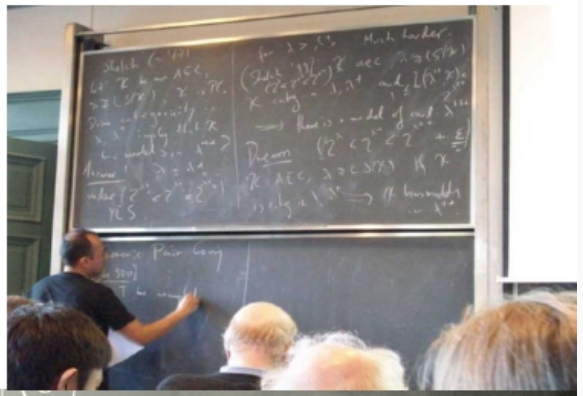
Project Topoi

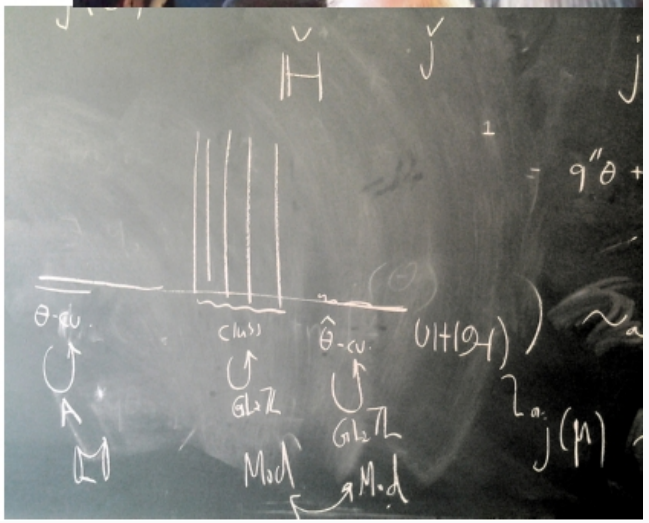
Entre agosto y noviembre de 2013 : 440 imágenes, *casi* sin texto. Aprox. 40 **topoi**



local/global – topoi, sheaves, space (A)

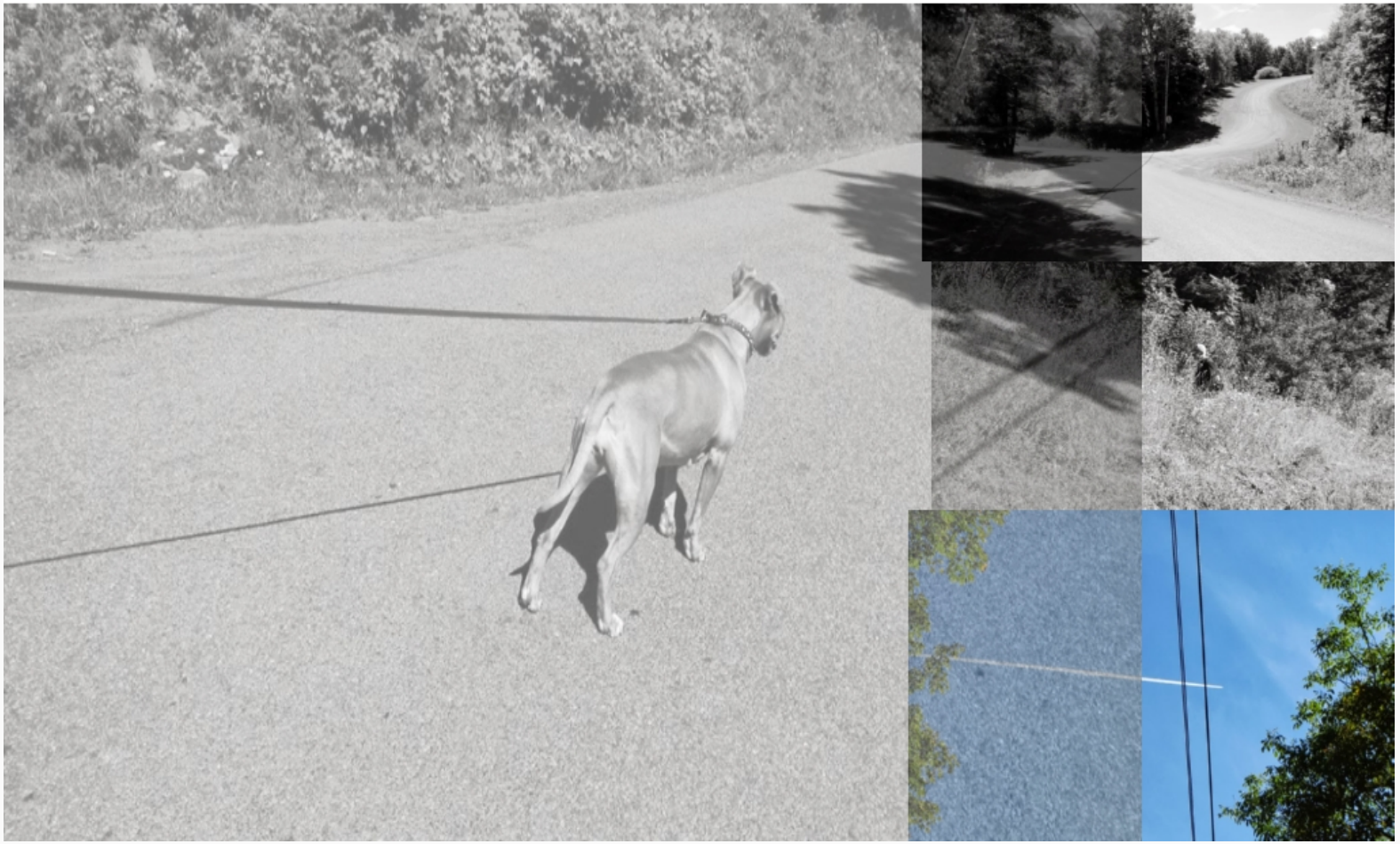






Project Topoi

"There" (W, in response to A)

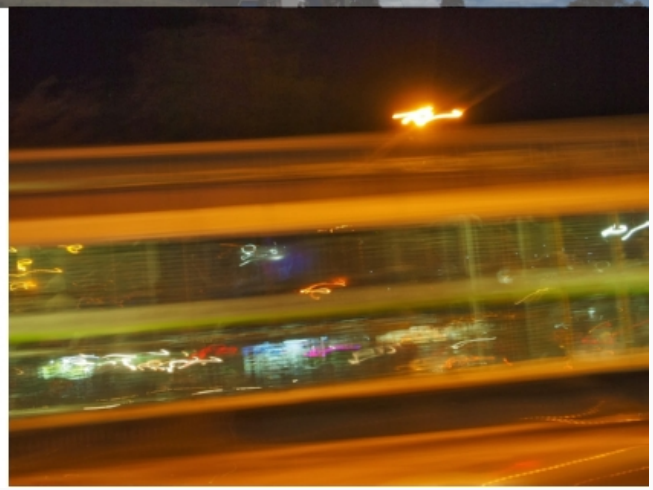


Project Topoi

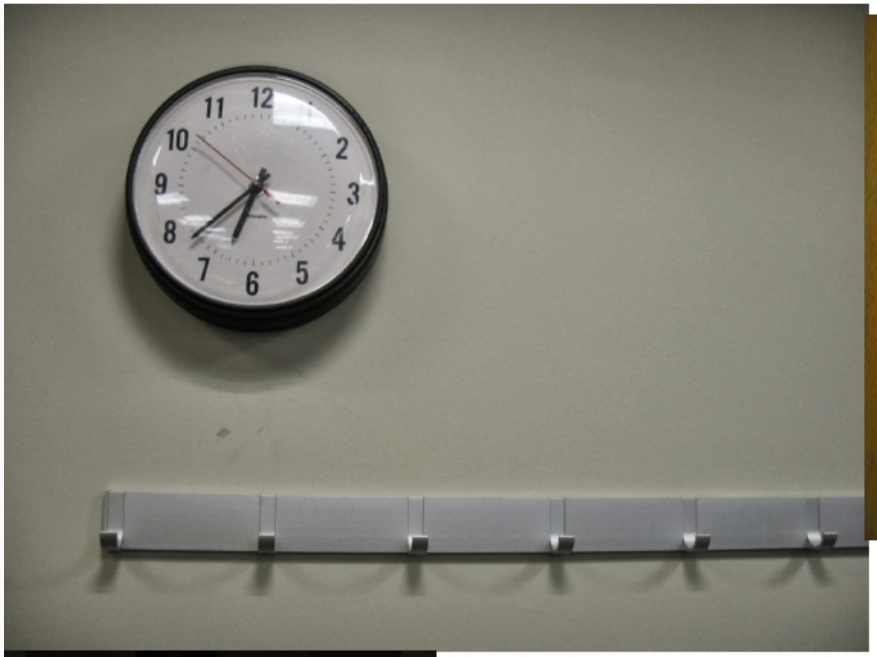
"Around" (response to A and W) - R

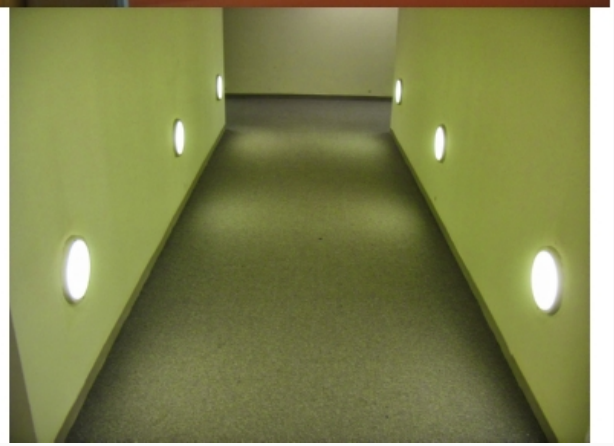
Project Topoi

(in response to R, W and A) in between - MC



R : academic topos I



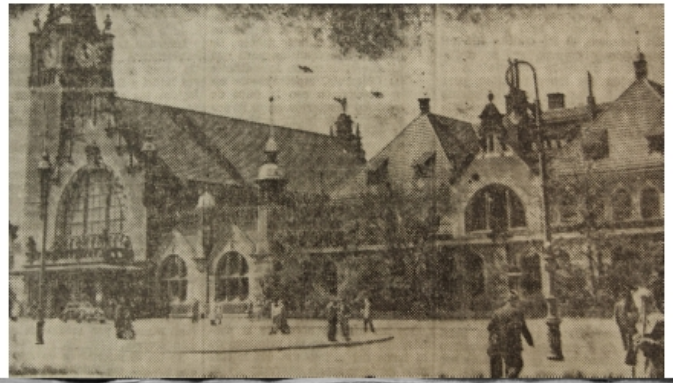


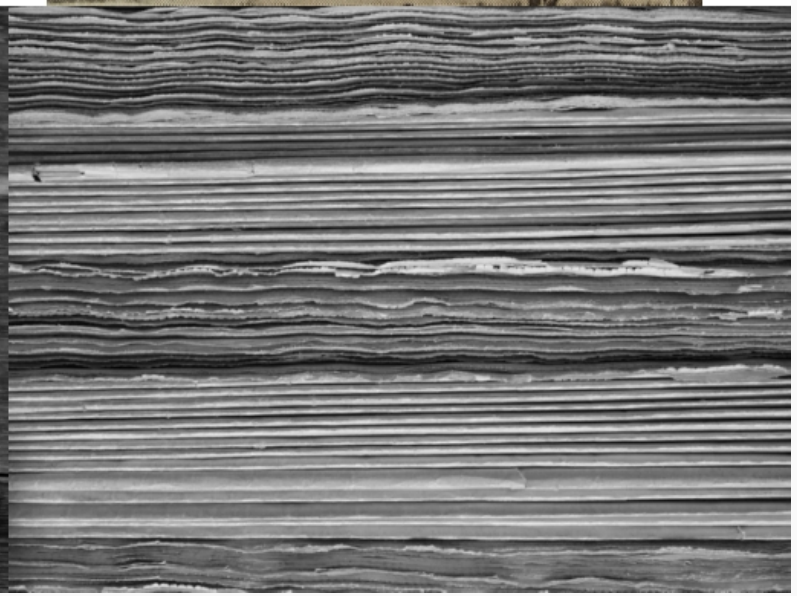
Project Topoi

A : (part of) mist, myster, mystest **topos**

Project Topoi

MC : tés mnémes topoi - (very academic)





Project Topoi

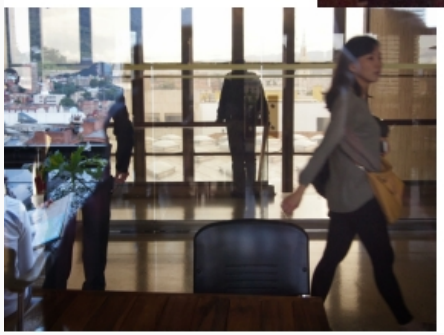
A : Blur topoi (in response to W)

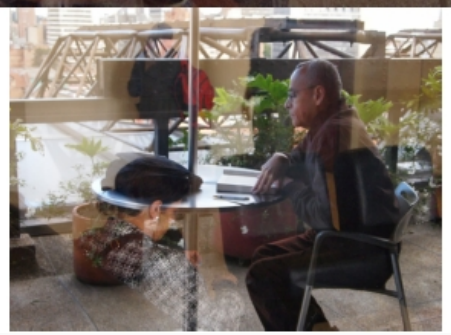
Project Topoi

In response to A (W's sheaves)

R : Undergraduate topos

MC : indiscrete - double topoi





Project Topoi

A - Throughness **topoi** (rites of passage)
- in response to MC's Cracked Topoi, and to Wanda's responses...

Project Topoi

W - Response to A: Throughness topoi (rites of passage)